GRAVITATIONAL WAVE INTERACTION WITH NORMAL AND SUPERCONDUCTING CIRCUITS

Pierluigi Fortini,[*] **Enrico Montanari**[*], **Antonello Ortolan**[#]

and

Gerhard Schäfer[◇]

Abstract The interaction, in the long–wavelength approximation, of normal and superconducting electromagnetic circuits with gravitational waves is investigated. We show that such interaction takes place by modifying the physical parameters R, L, C of the electromagnetic devices. Exploiting this peculiarity of the gravitational field we find that a circuit with two plane and statically charged condensers set at right angles can be of interest as a detector of periodic gravitational waves.

PACS numbers: 04.20

Annals of Physics **248**, 34–59 (1996)

[*] Department of Physics, University of Ferrara and INFN Sezione di Ferrara, I-44100 Ferrara, Italy.

[#] INFN National Laboratories of Legnaro, I-35020 Legnaro, Padova, Italy.

[◇] Max-Planck-Gesellschaft, AG Gravitationstheorie an der Friedrich-Schiller-Universität, D-07743 Jena, Germany.

1. Introduction

Future gravitational–wave astronomy requires gravitational–wave detectors of fantastic high sensitivity. Two designs of detectors are on the beam of realisation: bar detectors and laser–beam detectors. Both are promising to become enough sensitive for the detection and even for the detailed measurement of cosmic gravitational waves. Whereas the beam detectors will reach rather high sensitivity the bar detectors will be quite cheap. The sensitivity of the latter should be sufficient to detect gravitational waves from coalescing binaries in the VIRGO cluster and from type–II supernovae in our galaxy. Unfortunately, both of them happen not very often (about one per 30 years). However, there might exist other sources in our galaxy, like slightly deformed rotating neutron stars, in particular young asymmetric pulsars, or even completely unexpected sources which make gravitational–wave surveys in our galaxy, or other galaxies of the Local Group, very desirable. The centers of galaxies are very difficult to investigate with present–day telescopes so that our knowledge of the centers is very crude. Gravitational–wave detection might reveal interesting objects there.

Purpose of this paper is to work out in details a general formalism for treating the interaction of a gravitational wave with an electromagnetic (ohmic and superconducting) device in the quasi–stationary approximation. Such a formalism is in fact lacking and only partial cases are to be found in the literature (see ref. [1,3,11,16]).

As an application of this general theory we introduce and investigate a new design of detectors, normal and superconducting electromagnetic circuits which couple to gravitational waves in *non–parametric* manner. These devices are circuit–generalisations of a capacity–device approach invented by Mours and

Yvert in 1989. As we shall see in the following our devices could reach sensitivities comparable with the sensitivities of mechanical detectors for periodic radiation [20,21].

The paper is organized as follows. In Sect. 2 we discuss, in quasi–stationary approximation, the interaction of electromagnetic fields and currents with weak external gravitational fields. In particular we discuss under which conditions the conductors, i.e. the ions of the lattices of the conductors, can be treated as freely falling in external gravitational wave fields. Sect. 3 presents the general theory of normal conducting circuits in the field of external gravitational waves. In this section the parametric nature of the interaction is shown. Sect. 4 refines the results of Sect. 3 to superconducting circuits. In Sect. 5 the influence of dielectric and magnetic matter is investigated. Finally, in Sect. 6, we apply our theoretical results to concrete, thermal–noise–dominated detectors and derive sensitivity–limits for the detection of gravitational waves.

2. Electromagnetic Fields and Conductors in Curved Spacetime

The electromagnetic field equations and the equations of motion for charged matter (ions and electrons) in an external gravitational field $g_{\mu\nu}(\mu,\nu = 0,1,2,3)$ can be derived from the action

$$S = S_{em} + S_m + S_{int}, \tag{2.1}$$

where $S_{em} = (1/4\pi c)\int d^4x\sqrt{-g}\; F_{\mu\nu}F^{\mu\nu}$ $(g = \det(g_{\mu\nu}))$ is the action for the free electromagnetic field, S_m is the action for the conductors that we will discuss below, and $S_{int} = \dfrac{1}{c^2}\displaystyle\int d^4x\sqrt{-g}\; j^\mu A_\mu$, with the current density defined by

$$j^\mu = \frac{c\rho}{\sqrt{-g_{00}}}\frac{dx^\mu}{dx^0},$$

is the action for the coupling between the electromagnetic field and the charges and currents in the conductor. The electromagnetic field–strength tensor $F_{\mu\nu}$ is

related with the four–vector potential A_μ by $F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}$ and we have the following relation between $F_{\mu\nu}$ and the standard electromagnetic field–strength entities, E_i and B_i $(i = 1, 2, 3)$,

$$F_{\mu\nu} = \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & B_3 & -B_2 \\ E_2 & -B_3 & 0 & B_1 \\ E_3 & B_2 & -B_1 & 0 \end{pmatrix}. \tag{2.2}$$

By varying the action (2.1) with respect to A_μ we get the De Rahm equations. In the Lorentz gauge, $\nabla_\nu A^\nu = 0$, they read

$$\nabla^\nu \nabla_\nu A^\mu - A^\rho R_\rho^{\ \mu} = -\frac{4\pi}{c}\, j^\mu. \tag{2.3}$$

Eq. (2.3) generalizes the inhomogeneous Maxwell equations in the Lorentz gauge to curved spacetime; $R_{\mu\nu}$ is the Ricci tensor defined by

$$R_{\mu\nu} = R^\alpha_{\ \ \mu\alpha\nu},$$

where $R^\alpha_{\ \ \mu\beta\nu}$ is the Riemann tensor

$$R^\alpha_{\ \ \mu\beta\nu} = \frac{\partial \Gamma^\alpha_{\mu\nu}}{\partial x^\beta} - \frac{\partial \Gamma^\alpha_{\mu\beta}}{\partial x^\nu} + \Gamma^\alpha_{\eta\beta}\Gamma^\eta_{\mu\nu} - \Gamma^\alpha_{\eta\nu}\Gamma^\eta_{\mu\beta}, \tag{2.4}$$

and where $\Gamma^\alpha_{\mu\nu}$ denote the Christoffel symbols

$$\Gamma^\alpha_{\mu\nu} = \frac{1}{2} g^{\alpha\beta}\left(\frac{\partial g_{\beta\nu}}{\partial x^\mu} + \frac{\partial g_{\beta\mu}}{\partial x^\nu} - \frac{\partial g_{\mu\nu}}{\partial x^\beta}\right), \tag{2.5}$$

which enter in the definition of the covariant derivative ∇_μ (notations and conventions as in ref. [4] pp. 223–4).

Our aim is to study the interaction of a gravitational wave, emitted for instance by a cosmic source, with electromagnetic circuits. In the interaction region the metric tensor $g_{\mu\nu}$ satisfies the Einstein equations in vacuo, $R_{\mu\nu} = 0$. For weak gravitational fields, to which we shall limit ourselves in this paper, the metric tensor and its determinant can be written as

$$\begin{aligned} g_{\mu\nu} &= \eta_{\mu\nu} + h_{\mu\nu}, \\ \sqrt{-g} &= 1 + \frac{h}{2}, \end{aligned} \tag{2.6}$$

where $\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$ is the Minkowski metric tensor and where $h_{\mu\nu}$, with $|h_{\mu\nu}| << 1$, is its small perturbation; $h \equiv h^{\mu}_{\ \ \mu} = \eta^{\mu\nu} h_{\mu\nu}$. In this approximation we can fix a particular coordinate frame, respectively gauge, the so–called TT (transverse and traceless) gauge [22] specified by $h = 0$, $h_{0\mu} = 0$, and $\partial_j h^{ij} = 0$, where the solutions $h_{\mu\nu}$ of the homogeneous Einstein equations take the very simple form:

$$h_{ij} = A_+ (\boldsymbol{e}_+)_{ij} e^{i(k_\mu x^\mu + \varphi_+)} + A_\times (\boldsymbol{e}_\times)_{ij} e^{i(k_\mu x^\mu + \varphi_\times)}, \tag{2.7}$$

where $k^\mu \equiv (\omega_g/c, \boldsymbol{k})$ and $k^\mu k_\mu = 0$; $A_{+,\times}$ are the amplitudes of the two polarization states and $\boldsymbol{e}_{+,\times}$ are the two transverse ($e_{ij}k^j = 0$) and traceless ($e^i_{\ \ i} = 0$) polarization tensors [22].

Neglecting second order terms in $h_{\mu\nu}$ one can easily show that the linearized De Rahm equations are

$$\begin{aligned} &A^{\mu,\nu}_{\ \ \ \ ,\nu} + h^{\mu}_{\ \ \nu,\beta}\ A^{\beta,\nu} + h^{\mu}_{\ \ \beta,\nu}\ A^{\beta,\nu} - h_{\nu\beta}^{\ \ \ ,\mu}\ A^{\beta,\nu} - h^{\nu\alpha}\ A^{\mu}_{\ \ ,\alpha,\nu} = -\frac{4\pi}{c}\ j^\mu, \\ &\partial_\nu\ A^\nu = 0. \end{aligned} \tag{2.9}$$

We shall consider our electromagnetic system in the so–called long–wavelength (or quasi–stationary) approximation in which the frequency of the electromagnetic oscillations ν_{em} is much smaller than c/d where d is the typical linear dimension of the system. This condition is equivalent to the assumption that all time derivatives of the potential in (2.9) are negligible with respect to space derivatives and also that the total electromagnetic radiation of the system is negligibly small (this implies also $v/c << 1$ where v is the typical velocity of the charged particles in the system). If these conditions are satisfied it has been shown ([9] and e.g. [14] §75) that $S_{em} = -(1/2)S_{int}$ and thus, the total action reduces to

$$S = \frac{1}{2c^2} \int \sqrt{-g}\ d^4x\ j^\mu A_\mu + S_m. \tag{2.10}$$

In order to get a consistent picture of the dynamics of a non–radiating system we shall request that the reduced gravitational wavelength $\lambda_g/2\pi$ is much greater than d. This entails that the leading term in eq. (2.9) is the one proportional to h_{ij}. Within these approximations we get

$$A^{\mu,k}_{\ \ ,k} - h^{ij}\ A^{\mu}_{\ ,i,j} = -\frac{4\pi}{c}\ j^{\mu}. \tag{2.11}$$

We shall now investigate the matter action S_m. We shall show that if the elastic bodies forming the circuits can be treated as free falling in the field of the gravitational wave, we can discard the mechanical part of the circuit. It is a well known (see [22]) property of the TT system that if freely falling bodies are at rest before the arrival of the wave their spatial TT coordinates remain constant in time also in the oscillating gravitational field. This fact will greatly simplify our treatment as all the relevant circuit parameter perturbations can be found in TT coordinates by integration over unperturbed coordinate paths.

To find the free-falling conditions let us consider the normal modes of a solid body as a collection of non–coupled damped harmonic oscillators. This problem can be easily solved in the Fermi Normal Coordinates (FNC) frame (see [5]). The deformation from the equilibrium shape can be written as a sum

$$\delta \boldsymbol{x} = \sum_{n,l} A_{nl}(t)\boldsymbol{\psi}_{nl}(\boldsymbol{x}) \tag{2.12}$$

The l–label refers to the kind of modes: for instance, as far as cylindrical bars are concerned, it refers to the longitudinal, torsional and flexural modes. The n–label refers to the eigenfrequencies of a given mode.

For each specific value of l the fundamental mode shall be denoted by $n = 1$. In the following we assume that only one specific l–mode is strongly interacting with the gravitational wave. For instance, for a spherical body R_{nl} is different from zero only when $l = 2$.

In the presence of a gravitational wave, the coefficients A_{nl} are determined by the equation (see [4] eqs. (37.42b–c)):

$$\ddot{A}_{nl} + \frac{1}{\tau_{nl}}\dot{A}_{nl} + \omega_{nl}^2 A_{nl} = R_{nl}, \tag{2.13}$$

where

$$R_{nl} = \frac{1}{2}\frac{1}{M}\ddot{h}_{ij}\int \psi_{nl}^i x^j \rho d^3x \tag{2.14}$$

(ρ is the mass density of the body and M its total mass). The solution in Fourier space ($\xi(\omega) = (2\pi)^{-1/2}\int_{-\infty}^{+\infty}\xi(t)\exp(-i\omega t)\ dt$) is given by

$$A_{nl}(\omega) = -\frac{1}{2}t_{nl}^{ij}h_{ij}(\omega)\frac{\omega^2}{\omega_{nl}^2 - \omega^2 + i\omega/\tau_{nl}}, \tag{2.15}$$

where t_{nl}^{ij} is defined as

$$t_{nl}^{ij} = \frac{1}{M}\int \psi_{nl}^i x^j \rho d^3x. \tag{2.16}$$

For monochromatic gravitational waves of frequencies $\omega_g >> \omega_{1l}$ we get

$$A_{1l}(t) = \frac{1}{2}\ t_{1l}^{ij}\ h_{ij}(t) \tag{2.17}$$

that is to say, in its fundamental mode the body behaves as a freely moving system. This is the operating condition for the interferometric devices under construction in Europe (VIRGO) and in the United States (LIGO).

Let us now consider the opposite case where $\omega_g << \omega_{1l}$. We get

$$A_{nl}(t) = -\frac{1}{2}\ t_{nl}^{ij}\ h_{ij}(t)\left(\frac{\omega_g}{\omega_{nl}}\right)^2 \tag{2.18}$$

that is the body is practically at rest and the geometrical properties of the circuit do not change.

Finally at mechanical resonance of the fundamental mode $\omega_g = \omega_{1l}$ we get $A_{1l}(\omega) = it_{1l}^{ij}\ h_{ij}(\omega)\mathcal{Q}_{1l}$ where $\mathcal{Q}_{1l} = \pi\nu_{1l}\tau_{1l}$ is the quality factor of the oscillator. This is the operation condition for the resonant antennae already

working in Italy (Universities of Rome and National Laboratories of INFN at Frascati and Legnaro) and in the United States (Universities of Louisiana and Stanford).

Now, in order to find free falling conditions in our case, we examine in more details R_{nl}. Comparing eq. (37.45) with eq. (7b) Box 37.4 and taking into account definition (4) Box 37.4 of [4], it can be reasonably assumed that R_{nl} may have the following behaviour as a function of n

$$R_{nl} \simeq \frac{1}{n^2} R_{1l} \tag{2.19}$$

This implies that the energy deposited in the n–th normal mode at resonance is approximately independent on n (see [4] eq. (37.43)) if we assume $\omega_{nl} \simeq n\omega_{1l}$. With these assumptions the A_{nl} coefficients are given by

$$A_{nl}(\omega) = -\frac{1}{2}\frac{1}{n^2} t_{1l}^{ij} h_{ij}(\omega) \frac{\omega^2}{n^2\omega_{1l}^2 - \omega^2 + i\omega/\tau_{nl}} \tag{2.20}$$

If we now consider again an incoming gravitational wave with frequency $\omega_g >> \omega_{1l}$, different from all the other ω_{nl}, then from eq. (2.20) one can see that only the fundamental mode is practically excited. For instance if the first harmonic excited by the wave is $n = 3$ (see, e.g. [4], eq. (37.45)) then $A_{3l} \simeq \frac{1}{10} A_{1l}$. Therefore the body is free falling because it follows the incoming gravitational wave like a cloud of dust (see eq. (2.17)) and so it is practically at rest in TT gauge. One can see therefore that, by suitably arranging the fundamental mode frequency ω_{1l} with respect to the frequency chosen for observing gravitational radiation, the condition of free falling can be easily fulfilled. In the rest of this paper we shall restrict ourselves to physical devices where these conditions are satisfied and therefore we neglect the matter–action term in (2.1).

3. General Theory of Electromagnetic Circuits

To establish the behaviour of a circuit in the field of a weak gravitational wave we generalize the lagrangian approach of [15]. Let γ_a ($a = 1, 2, \ldots, N$) be a system of N conducting non–ferromagnetic one dimensional (wires) or extended bodies (capacitors). The quasi–stationary approximation allows us to neglect surface currents in extended conductors and charged wires. The charge over conductor surface and the current flowing in wires can be treated as not depending on spatial coordinates. The relations between, on the one side, j^μ and the current I and, on the other side, charge density ρ and the charge dQ in an infinitesimal volume element can be written in any coordinate system as (see for instance [14] §90)

$$\left\{ \begin{array}{l} I dx^i = j^i \sqrt{-g}\ d^3x \\ \\ dQ = \rho \sqrt{\,^3 g}\ d^3x \end{array} \right. , \tag{3.1}$$

where ^3g is the determinant of the three–dimensional metric $^3g_{ij} = g_{ij} - g_{0i}g_{0j}/g_{00}$. The equations for I and Q can be found by substituting in the action (2.10) the (formal) solution of the De Rahm equation (2.3) for gravitational waves. If we can neglet $g_{\mu\nu,\alpha}$ with respect to $g_{\mu\nu}$ (i.e. for slowly varying gravitational fields) then $\Box \equiv \nabla^\mu \nabla_\mu \approx g^{\alpha\beta}\partial_\alpha\partial_\beta$. In this case we can write

$$A_\mu(x) = \frac{1}{c}\int_\Omega \overline{K}_{\mu\nu}[x, x'] j^\nu(x') \sqrt{-g(x')}\ d^4x', \tag{3.2}$$

where $\overline{K}_{\mu\nu}$ is the retarded Green function, solution of the equation

$$\Box_x\ \overline{K}_{\mu\nu}[x, x'] = -4\pi \frac{g_{\mu\nu}(x')}{\sqrt{-g(x')}} \delta^4(x^\alpha - x'^\alpha) \tag{3.3}$$

In a synchronous reference frame $x \equiv (ct, \boldsymbol{x})$, where $^3g_{ij} = g_{ij}$ and $-g = \,^3g$, we can integrate eq. (3.2) over the time coordinate x'^0 and get the generalized retarded potential ([4] p. 500)

$$A_\mu(\boldsymbol{x}, t) = \frac{1}{c}\int_V K_{\mu\nu}[x, \boldsymbol{x}'] j^\nu(x^0, \boldsymbol{x}') \sqrt{\,^3g(x^0, \boldsymbol{x}')} d^3x'. \tag{3.4}$$

Now we have to introduce the lagrangian $\mathcal{L}$ of a system. ¿From (2.10) and (3.4) we obtain the total lagrangian of a system of conductors and wires in curved spacetime

$$\mathcal{L} = \frac{1}{2c^2} \int_V \sqrt{{}^3g(x)}\; d^3x \int_V \sqrt{{}^3g(x^0, \boldsymbol{x}')}\; d^3x' \times \\ \times\; K_{\mu\nu}[x, \boldsymbol{x}']\; j^\mu(x)\; j^\nu(x^0, \boldsymbol{x}') \tag{3.5}$$

¿From the relations (3.1), taking into account that I and Q are only functions of time in the quasi–stationary–field approximation and that $\overline{K}^{0i} = 0$ in synchronous reference frames, we get

$$\mathcal{L} = \sum_{a,b=1}^{N} \frac{I_{\gamma_a}\; I_{\gamma_b}}{2c^2} \int_{\gamma_a} dx^i \int_{\gamma_b} dx'^j K_{ij}[x, \boldsymbol{x}'] \\ + \frac{1}{2} \int_{\gamma_a} dQ \int_{\gamma_b} dQ' K_{00}[x, \boldsymbol{x}']. \tag{3.6}$$

For quasi–stationary fields charges are distributed over the conductor surfaces A_{γ_a} and so we can replace in (3.6) the charge integrals with surface integrals. This leads to

$$\mathcal{L} = \sum_{a,b=1}^{N} \left[\frac{1}{2}\; L_{\gamma_a\gamma_b}(t) I_{\gamma_a}\; I_{\gamma_b} - \frac{1}{2}\; \widehat{C}_{\gamma_a\gamma_b}(t) Q_{\gamma_a} Q_{\gamma_b} \right], \tag{3.7}$$

where $L_{\gamma_a\gamma_b}$ are the generalized coefficients of mutual inductance

$$L_{\gamma_a\gamma_b}(t) = \frac{1}{c^2} \int_{\gamma_a} dx^i \int_{\gamma_b} dx'^j K_{ij}[x, \boldsymbol{x}'], \tag{3.8}$$

and $\widehat{C}_{\gamma_a\gamma_b}$

$$Q_{\gamma_a} \widehat{C}_{\gamma_a\gamma_b}(t) Q_{\gamma_b} = - \int_{\gamma_a} dA_{\gamma_a} \int_{\gamma_b} dA'_{\gamma_b} \frac{dQ_{\gamma_a}}{dA_{\gamma_a}} K_{00}[x, \boldsymbol{x}'] \frac{dQ_{\gamma_b}}{dA'_{\gamma_b}}, \tag{3.9}$$

define univocally the coefficients of capacity $C_{\gamma_a\gamma_b}$ by means of the equations $\sum_{b=1}^{N} \widehat{C}_{\gamma_a\gamma_b} C_{\gamma_b\gamma_c} = \delta_{ac}$. ¿From the above formulae one easily recognizes that

the action of gravitational fields in synchronous reference frames (such as TT–gauge frames) is a change in the geometrical circuit parameters, i.e. the interaction of gravitational radiation with standard electromagnetic circuits is of parametric nature.

In order to take into account also dissipations of real circuits we go over to Ohm's law written in its covariant form ([8] p. 263); for the current in wires one has

$$j^\alpha - j_\beta \; u^\beta \; u^\alpha = \sigma \; F^\alpha_{\;\;\nu} \; u^\nu \;, \tag{3.10}$$

where u^ν is the normalized four velocity of the lattice of the conductor γ_a, and σ_a is its conductivity which we assume constant in the range of frequencies we are interested in. As the lattice has fixed TT coordinates $u^\mu = \delta^\mu_0$, the total dissipated power in our circuits by Joule effect is

$$\begin{aligned} P &\equiv \int_V \sqrt{{}^3 g} \; d^3x \; j^\mu \; F_{\mu\nu} \; u^\nu \\ &= \sum_a \frac{1}{\sigma_a} \int_{\gamma_a} \sqrt{{}^3 g} \; d^3x \; g_{ij} \; j^i(x^k) \; j^j(x^k), \end{aligned} \tag{3.11}$$

which can be rewritten in the form

$$P = \sum_{a=1}^{N} R_{\gamma_a}(t) I^2_{\gamma_a}, \tag{3.12}$$

where $R_a(t)$ are generalized resistances

$$R_{\gamma_a}(t) = \frac{g_{ij}}{\sqrt{{}^3 g}} \int_{\gamma_a} \frac{1}{\sigma_{\gamma_a} S_{\gamma_a}} \; t^j_{\gamma_a} \; dx^i; \tag{3.13}$$

here $t^i_{\gamma_a}$ is the unit vector (${}^3g_{ij} t^i_{\gamma_a} \; t^j_{\gamma_a} = 1$) tangent to the wire γ_a and $dS_{\gamma_a} \equiv d^3x_{\gamma_a}/dl$, ($dl^2 = g_{ij} dx^i dx^j$) is the cross–section of the wire.

In order to get the Euler–Lagrange equations from the lagrangian (3.7) we have to set up a relation among our charges and currents. ¿From (3.1) and the charge conservation relation follows $I_{\gamma_a} = \dot{Q}_{\gamma_a}$. Hereof the equations of motion

result in the form

$$\sum_{b=1}^{N}\Big[L_{\gamma_a\gamma_b}(t)\ \ddot{Q}_{\gamma_b}+\dot{L}_{\gamma_a\gamma_b}(t)\ \dot{Q}_{\gamma_b}+\widehat{C}_{\gamma_a\gamma_b}(t)\ Q_{\gamma_b}\Big]=0. \tag{3.14}$$

Once we have defined the dissipation function $\Phi = P/2$ we can substitute the r.h.s. of the equation (3.14) with $\partial\Phi/\partial\dot{Q}_{\gamma_a}$ and write (see [13] §1.5)

$$\sum_{b=1}^{N} L_{\gamma_a\gamma_b}(t)\ddot{Q}_{\gamma_b}+\dot{L}_{\gamma_a\gamma_b}(t)\dot{Q}_{\gamma_b}+\widehat{C}_{\gamma_a\gamma_b}(t)\ Q_{\gamma_b}=-R_{\gamma_a}(t)\dot{Q}_{\gamma_a}. \tag{3.15}$$

These equations describe, in TT coordinates, RLC circuits in external gravitational radiation fields when dissipations are taken into account.

In the weak gravitational field approximation one can easily show (cf. [10]) that

$$K_{\mu\nu}=\frac{\eta_{\mu\nu}+h_{\mu\nu}}{|\boldsymbol{x}-\boldsymbol{x}'|}+\frac{1}{2}h_{kl}\partial'^{k}\partial'^{l}|\boldsymbol{x}-\boldsymbol{x}'|\eta_{\mu\nu} \tag{3.16}$$

satisfies eq. (2.11) through eq. (3.4), and therefore it gives the right expression of the Green function $K_{\mu\nu}$ in the linear and quasi–static field approximation and for a TT–metric perturbation $h_{\mu\nu}(t)$. From eqs. (3.8), (3.9) and (3.13) it follows that, in the same approximation, the coefficients $L_{\gamma_a\gamma_b}(t)$, $\widehat{C}_{\gamma_a\gamma_b}(t)$ and $R_{\gamma_a}(t)$ read

$$\begin{aligned} L_{\gamma_a\gamma_b}(t) &= {}^0L_{\gamma_a\gamma_b}+l_{\gamma_a\gamma_b}(t)\\ \widehat{C}_{\gamma_a\gamma_b}(t) &= {}^0\widehat{C}_{\gamma_a\gamma_b}+\widehat{c}_{\gamma_a\gamma_b}(t)\\ R_{\gamma_a}(t) &= {}^0R_{\gamma_a}+r_{\gamma_a}(t)\end{aligned} \tag{3.17}$$

where ${}^0L_{\gamma_a\gamma_b}$, ${}^0\widehat{C}_{\gamma_a\gamma_b}$, and ${}^0R_{\gamma_a}$ are respectively the usual inductance, capacitance, and resistance coefficients in flat spacetime, defined as

$${}^0L_{\gamma_a\gamma_b}=\frac{1}{c^2}\int_{\gamma_a}\int_{\gamma_b}\delta_{ij}\frac{dx^i_{\gamma_a}dx'^j_{\gamma_b}}{|\boldsymbol{x}_{\gamma_a}-\boldsymbol{x}'_{\gamma_b}|}, \tag{3.18}$$

$${}^0R_{\gamma_a}=\delta_{ij}\int_{\gamma_a}\frac{t^i_{\gamma_a}t^j_{\gamma_a}}{\sigma_{\gamma_a}}\frac{dl}{dA_{\gamma_a}} \tag{3.19}$$

$$Q^{\gamma_a}\ {}^0\hat{C}_{\gamma_a\gamma_b}Q^{\gamma_b} = \int_{\gamma_a}\int_{\gamma_b} \frac{dQ^{\gamma_a}}{dA_{\gamma_a}} \frac{dA_{\gamma_a}dA'_{\gamma_b}}{|\boldsymbol{x}_{\gamma_a} - \boldsymbol{x}'_{\gamma_b}|} \frac{dQ^{\gamma_b}}{dA'_{\gamma_b}}. \tag{3.20}$$

Here $l_{\gamma_a\gamma_b}(t)$, $\widehat{c}_{\gamma_a\gamma_b}(t)$ and $r_{\gamma_a}(t)$ are the time dependent perturbations induced by the gravitational waves on the circuit parameters which can be written as:

$$\begin{aligned} l_{\gamma_a\gamma_b} =& h_{ij}\lambda^{ij}_{\gamma_a\gamma_b} \\ \hat{c}_{\gamma_a\gamma_b} =& h_{ij}\chi^{ij}_{\gamma_a\gamma_b} \\ r_{\gamma_a} =& h_{ij}\varrho^{ij}_{\gamma_a} \end{aligned} \tag{3.21}$$

with

$$\lambda^{ij}_{\gamma_a\gamma_b} = \frac{1}{c^2}\int_{\gamma_a}\int_{\gamma_b} \frac{dx^i_{\gamma_a}dx'^j_{\gamma_b}}{|\boldsymbol{x}_{\gamma_a} - \boldsymbol{x}'_{\gamma_b}|} + \frac{1}{2c^2}\int_{\gamma_a}\int_{\gamma_b} \delta_{kl}\partial'^i\partial'^j|\boldsymbol{x}_{\gamma_a} - \boldsymbol{x}'_{\gamma_b}|dx^k_{\gamma_a}dx'^l_{\gamma_b}, \tag{3.22}$$

$$Q^{\gamma_a}\chi^{ij}_{\gamma_a\gamma_b}Q^{\gamma_b} = \frac{1}{2}\int_{\gamma_a}\int_{\gamma_b} \frac{dQ^{\gamma_a}}{dA_{\gamma_a}}\frac{dQ^{\gamma_b}}{dA'_{\gamma_b}}\partial'^i\partial'^j|\boldsymbol{x}_{\gamma_a} - \boldsymbol{x}'_{\gamma_b}|dA_{\gamma_a}dA'_{\gamma_b} \tag{3.23}$$

and

$$\varrho^{ij}_{\gamma_a} = \int_{\gamma_a} \frac{t^i_{\gamma_a}t^j_{\gamma_a}}{\sigma_{\gamma_a}}\frac{dl}{dS_{\gamma_a}}. \tag{3.24}$$

The coefficients $\lambda^{ij}_{\gamma_a\gamma_b}$, $\chi^{ij}_{\gamma_a\gamma_b}$ and $\varrho^{ij}_{\gamma_a}$, defined in (3.22), (3.23) and (3.24), are pure geometric quantities which play a similar rôle as the mass-quadrupole tensor $Q^{ij} = \int_V \rho(\boldsymbol{x})(x^ix^j - (1/3)\delta^{ij}x^kx_k)\ d^3x$ in the mechanical interaction between a gravitational wave and a solid body.

We notice that the equations that describe the interaction between gravitational waves and electromagnetic circuits are parametric ones. This means that gravity is not acting as electromotoric force but is changing the circuit parameters. The effect of the small perturbation of the parameters of the circuit is a frequency and amplitude modulation (see for instance [18]).

4. The superconducting circuit

Below a certain temperature T_t, called transition temperature, some metals become superconducting. The most impressive properties of this state are

the sudden drop of the electrical resistance and the Meissner effect. F. London in 1935 (cfr. [12]) suggested a macroscopic theory of the pure superconducting state which unified these two experimental facts under the same theoretical scheme. He supposed that the density current flowing in the superconductor was a sum of two currents: the normal $\boldsymbol{j}_n$ and the supercurrent $\boldsymbol{j}_s$

$$\boldsymbol{j} = \boldsymbol{j}_s + \boldsymbol{j}_n. \tag{4.1}$$

The two kinds of current are related to the electromagnetic field inside the conductor in different ways. The supercurrent satisfies the two London equations

$$\begin{aligned} curl\ (\Lambda \boldsymbol{j}_s) &= -\frac{\boldsymbol{B}}{c} \\ \frac{\partial}{\partial t}\ (\Lambda \boldsymbol{j}_s) &= \boldsymbol{E}. \end{aligned} \tag{4.2}$$

Here Λ is a constant characteristic of the superconductor (its order of magnitude is $> 10^{-31}\ sec^2$); $\boldsymbol{B}$ and $\boldsymbol{E}$ are the intensities of the magnetic and electric fields. The normal current is connected with the electric field by Ohm's law

$$\boldsymbol{j}_n = \sigma \boldsymbol{E} \tag{4.3}$$

where σ is a continuous function of the temperature and has no jump at $T = T_t$. The total current $\boldsymbol{j}$ satisfies the Maxwell equations and the relation between the two currents turns out to be

$$\frac{\partial}{\partial t}\ \boldsymbol{j}_s = \frac{1}{\sigma\Lambda}\ \boldsymbol{j}_n = \beta\ \boldsymbol{j}_n \tag{4.4}$$

where β has the dimension of a frequency ($\beta = 1/\sigma\Lambda \approx 10^{12}\ Hz$). If the electromagnetic field oscillates at a frequency ω_{em} we have

$$|\boldsymbol{j}_n| = \frac{\omega_{em}}{\beta}\ |\boldsymbol{j}_s| \tag{4.5}$$

and if $\omega_{em} << \beta$ then $|\boldsymbol{j}_n| << |\boldsymbol{j}_s|$; in this way the sudden drop of the resistance is due only to the fact that the normal current (which is the one that dissipates)

becomes practically zero. Moreover, from eq. (4.2) and the Maxwell equations follows

$$\Delta B_i = \frac{1}{\lambda^2}\ B_i \tag{4.6}$$

where

$$\lambda = c\ \sqrt{\frac{\Lambda}{4\pi}} \approx 10^{-5}\ cm \tag{4.7}$$

is the so called penetration depth. The regular solutions of the eq. (4.6) decrease exponentially from the surface of the superconductor to its interior with typical length of the order of the penetration depth. Therefore inside large bodies (large compared with the penetration depth) we have $\boldsymbol{B} = 0$.

The London theory of superconductivity is enough for our purposes. In fact, as in the preceeding section we didn't need a microscopical theory of conductivity so now we need only the knowledge of the macroscopical behaviour of a superconductor that is given by the eqs. (4.1)–(4.3).

In the following we point out the differences from the ohmic case. The first step is to write down the equation of an oscillating superconducting RLC circuit in flat spacetime. ¿From the Maxwell equations we get the energy theorem as usual

$$div\frac{c}{4\pi}(\boldsymbol{E}\times\boldsymbol{B}) + \frac{\partial}{\partial t}\frac{1}{8\pi}(\boldsymbol{B}^2 + \boldsymbol{E}^2) = -(\boldsymbol{j}\boldsymbol{E}). \tag{4.8}$$

For quasi–stationary conditions the first term is negligible. $(1/8\pi)(\boldsymbol{B}^2 + \boldsymbol{E}^2)$ is the energy density of the electromagnetic field and it has the same expression as in the ohmic case.

The differences come out when we write explicitly the work done by the electric field (see [12] §9),

$$(\boldsymbol{j}\boldsymbol{E}) = \frac{\partial}{\partial t}\left(\frac{1}{2}\Lambda\boldsymbol{j}_s^2\right) + \frac{1}{\sigma}\boldsymbol{j}_n^2. \tag{4.9}$$

The term $\frac{1}{2}\Lambda\boldsymbol{j}_s^2$ represents a reversible work (it is in fact the kinetic energy density of the supercurrent) while $(1/\sigma)\boldsymbol{j}_n^2$ is the energy density dissipated by

the normal current (Joule effect). Another difference arises from the fact that if $\omega_{em} << \beta$ the current is different from zero only for a few penetration depths below the surface. Therefore we need to be careful when we calculate the kinetic energy and the dissipated power. In fact, let us approximate the current as

$$\boldsymbol{j}(r,t) \approx \boldsymbol{j}(t)\ exp\left[-\frac{(r_0-r)}{\lambda}\right] \tag{4.10}$$

where r_0 is the radius of the wire ($r_0 >> \lambda$); the power lost as Joule heat is given then by

$$P = \int_V \frac{1}{\sigma}\ \boldsymbol{j}_n^2\ dV = R_s I_n^2 \tag{4.11}$$

where

$$I_n \approx 2\pi\lambda r_0\ |\boldsymbol{j}_n(t)| \tag{4.12}$$

and

$$R_s = \int_\gamma \frac{dl}{\sigma\eta}. \tag{4.13}$$

η is the effective section in which the current flows. Is takes the value

$$\eta \approx 4\pi\lambda r_0. \tag{4.14}$$

In a wire one has $R_s \approx (r_0/\lambda)\ R$, where R is the usual resistance. For the kinetic energy we get

$$W_{kin} = \int_V \frac{1}{2}\Lambda\ \boldsymbol{j}_s^2\ dV = \frac{1}{2\beta}\ R_s I_s^2 \tag{4.15}$$

where for the last equality see eq. (4.4); I_s is defined analogously to I_n.

Therefore we can write the lagrangian of an RLC superconducting circuit as ($I = I_s + I_n$, $Q = Q_s + Q_n$)

$$\mathcal{L} = \frac{1}{2}\left(L + \frac{R_s}{\beta}\right) I_s^2 + L I_s I_n + \frac{1}{2} L I_n^2 - \frac{1}{2C} Q_s^2 - \frac{1}{C} Q_s Q_n - \frac{1}{2C} Q_n^2. \tag{4.16}$$

The dissipation function reads

$$\Phi = \frac{1}{2} R_s I_n^2 \tag{4.17}$$

and the relation between I_n and I_s is given by (see eq. (4.4))

$$\dot{I}_s = \beta I_n. \tag{4.18}$$

Using the method of Lagrange multipliers (see for instance [13] §2.4) we find

$$\ddot{Q} + 2\gamma_s \dot{Q}_n + \omega_0^2 Q = 0 \tag{4.19}$$

where $2\gamma_s = R_s/L$ and $\omega_0^2 = 1/LC$. The only difference from the equation of an ohmic circuit is in the resistance term: in the superconducting case this term is only due to a part of the current (the normal current).

Making use of relation (4.18) we find for the total current a differential equation of the third order in time

$$\dddot{I} + \beta(1 + 2\Gamma)\ddot{I} + \omega_0^2 \dot{I} + \beta\omega_0^2 I = 0 \tag{4.20}$$

where $2\Gamma = R_s/\beta L << 1$. The solution of this equation can be written, up to terms of the first order in Γ, as

$$I(t) = Ae^{-\beta(1+2\Gamma)t} + e^{-\gamma_0 t}[B\cos(\Omega_0 t) + D\sin(\Omega_0 t)] \tag{4.21}$$

where

$$\begin{aligned} \Omega_0 =& \omega_0(1 - \Gamma) \\ \gamma_0 =& \frac{\Gamma\Omega_0^2}{\beta} \end{aligned} \tag{4.22}$$

The first term is a sort of opening extracurrent which has its origin when, after starting the current with an external e.m.f. of frequency ω_{emf}, we let oscillate the circuit freely. The coefficient A is practically different from zero only if ω_{emf} is very different from ω_0. In this case the current must change its frequency

according to (4.18). The characteristic time of this term, being of the order of β^{-1}, is too short to be taken into account in a macroscopical theory like this. Therefore we can neglect this term and put $A = 0$. In this case it is easy to recognize that the following equation

$$\ddot{I} + 2\gamma_0 \dot{I} + \Omega_0^2 I = 0 \tag{4.23}$$

has the same solution of eq. (4.20) i.e. an oscillating RLC superconducting circuit is equivalent to a normal one with resonance frequency Ω_0 and resistance $R_{eff} = R_s \omega_0^2/\beta^2$.

For N superconducting wires or bodies the electric and magnetic part of the lagrangian as well as eqs. (3.8) and (3.9) are the same as in the ohmic case. Without dissipation and superconducting kinetic contribution, the lagrangian in curved spacetime is still given by eq. (3.7). As far as the dissipation function is concerned, recalling that only the normal currents dissipate and that they flow only near the surface, we get

$$\Phi = \frac{1}{2} \sum_{a=1}^{N} R_{s\gamma_a}(t) I_{n\gamma_a}^2 \tag{4.24}$$

where

$$R_{s\gamma_a}(t) = \frac{g_{ij}}{\sqrt{^3 g}} \int_{\gamma_a} \frac{dx^i t^j_{\gamma_a}}{\sigma_{\gamma_a} \eta_{\gamma_a}} \tag{4.25}$$

in which η_{γ_a} is the effective cross–section (4.14) of the a–th wire. These equations allow us to write down the generalization of the kinetic energy

$$W_{kin}(t) = \frac{1}{2\beta} \sum_{a=1}^{N} R_{s\gamma_a}(t) I_{s\gamma_a}^2; \tag{4.26}$$

and finally the lagrangian of a system of N superconducting wires or bodies can be written as

$$\mathcal{L} = \sum_{a,b=1}^{N} \left[\frac{1}{2}\, L_{\gamma_a\gamma_b}(t) I_{\gamma_a}\, I_{\gamma_b} - \frac{1}{2}\, \widehat{C}_{\gamma_a\gamma_b}(t) Q_{\gamma_a} Q_{\gamma_b} \right] + \frac{1}{2\beta} \sum_{a=1}^{N} R_{s\gamma_a}(t) I_{s\gamma_a}^2. \tag{4.27}$$

The dissipation function is given by (4.24). To get the Euler-Lagrange equations from lagrangian the (4.27) and the dissipation function (4.24) we have to know the relations between charge and current and between charge and supercurrent. For a set of RLC circuits we have $I_{\gamma_a} = \dot{Q}_{\gamma_a}$ and because of the general covariant form of eq. (4.2) ($d/d\tau[\Lambda(j_s^\mu - j_s^\alpha u_\alpha u^\mu)] = F^\mu_\nu u^\nu$), the relation between Q_{sa} and Q_{na} is given in the curved case also by

$$\dot{Q}_{s\gamma_a} = \beta Q_{n\gamma_a}. \tag{4.28}$$

Using the method of Lagrange multipliers we get from (4.27) and (4.24)

$$\sum_{b=1}^{N} \Big[L_{\gamma_a\gamma_b}(t)\ddot{Q}_{\gamma_b} + \dot{L}_{\gamma_a\gamma_b}(t)\dot{Q}_{\gamma_b} + \widehat{C}_{\gamma_a\gamma_b}(t)\ Q_{\gamma_b}\Big] = -R_{s\gamma_a}(t)\dot{Q}_{n\gamma_a} \tag{4.29}$$

and by means of (4.28) we get the equations for the supercurrent

$$\begin{aligned}\sum_{b=1}^{N} \Big[L_{\gamma_a\gamma_b}(t)\ \dddot{Q}_{\ s\gamma_b} + (\beta L_{\gamma_a\gamma_b}(t) + \dot{L}_{\gamma_a\gamma_b}(t))\ddot{Q}_{s\gamma_b} + R_{s\gamma_a}(t)\ddot{Q}_{s\gamma_a} + \\ + \widehat{C}_{\gamma_a\gamma_b}(t)\dot{Q}_{s\gamma_b} + \beta\widehat{C}_{\gamma_a\gamma_b}(t)\ Q_{s\gamma_b}\Big] = 0.\end{aligned} \tag{4.30}$$

5. Conductors in the presence of matter

In this section we generalize our theory to the case in which the capacitor is filled by a dielectric and the inductance has a magnetic kernel. Within the usual approximations the matter can be characterized by a dielectric constant ϵ and a magnetic permeability μ which enter into the constitutive equations (see [8] p. 263):

$$\begin{gathered} H_{\alpha\beta}\ u^\alpha = \epsilon F_{\alpha\beta}\ u^\alpha \\ (g_{\alpha\beta}F_{\gamma\delta} + g_{\alpha\gamma}F_{\delta\beta} + g_{\alpha\delta}F_{\beta\gamma})\ u^\alpha = \mu\ (g_{\alpha\beta}H_{\gamma\delta} + g_{\alpha\gamma}H_{\delta\beta} + g_{\alpha\delta}H_{\beta\gamma})\ u^\alpha \end{gathered} \tag{5.1}$$

where u^α is the macroscopic 4–velocity of the medium. With these definitions the Maxwell equations become

$$\begin{gathered} F_{\mu\nu,\sigma} + F_{\nu\sigma,\mu} + F_{\sigma\mu,\nu} = 0 \\ H^{\mu\nu}{}_{;\nu} = \frac{4\pi}{c} j^\mu. \end{gathered} \tag{5.2}$$

Under free–falling conditions $u^\alpha = \delta^\alpha_0$, eqs. (5.1) reduce to

$$H_{0k} \;=\; \epsilon\; F_{0k} \qquad\qquad F_{ik} \;=\; \mu\; H_{ik}. \tag{5.3}$$

In this way the second pair of Maxwell equations becomes

$$\begin{aligned} F^{0\nu}{}_{;\nu} &= \frac{4\pi}{\epsilon}\,\frac{j^0}{c} \\ F^{k\nu}{}_{;\nu} &= \frac{4\pi}{c}\;\mu\; j^k. \end{aligned} \tag{5.4}$$

The action of the free electromagnetic field reads $S_{em} = (1/4\pi c)\int d^4x\sqrt{-g}\; H^{\mu\nu}F_{\mu\nu}$.

With these notations eqs. (3.18) and (3.22) become (see [15] §§ 30-33)

$$^0L_{\gamma_a\gamma_b} = \frac{\mu}{c^2}\int_{\gamma_a}\int_{\gamma_b}\delta_{ij}\,\frac{dx^i_{\gamma_a}dx'^j_{\gamma_b}}{|\boldsymbol{x}_{\gamma_a}-\boldsymbol{x}'_{\gamma_b}|}, \tag{5.5}$$

$$\lambda^{ij}_{\gamma_a\gamma_b} = \frac{\mu}{c^2}\int_{\gamma_a}\int_{\gamma_b}\frac{dx^i_{\gamma_a}dx'^j_{\gamma_b}}{|\boldsymbol{x}_{\gamma_a}-\boldsymbol{x}'_{\gamma_b}|} + \frac{\mu}{2c^2}\int_{\gamma_a}\int_{\gamma_b}\delta_{kl}\partial'^i\partial'^j|\boldsymbol{x}_{\gamma_a}-\boldsymbol{x}'_{\gamma_b}|dx^k_{\gamma_a}dx'^l_{\gamma_b}, \tag{5.6}$$

while (3.20) and (3.23) become

$$Q^{\gamma_a}\;{}^0\hat{C}_{\gamma_a\gamma_b}Q^{\gamma_b} = \frac{1}{\epsilon}\int_{\gamma_a}\int_{\gamma_b}\frac{dQ^{\gamma_a}}{dA_{\gamma_a}}\,\frac{dA_{\gamma_a}dA'_{\gamma_b}}{|\boldsymbol{x}_{\gamma_a}-\boldsymbol{x}'_{\gamma_b}|}\,\frac{dQ^{\gamma_a}}{dA'_{\gamma_b}}. \tag{5.7}$$

$$Q^{\gamma_a}\chi^{ij}_{\gamma_a\gamma_b}Q^{\gamma_b} = \frac{1}{2\epsilon}\int_{\gamma_a}\int_{\gamma_b}\frac{dQ^{\gamma_a}}{dA_{\gamma_a}}\frac{dQ^{\gamma_b}}{dA'_{\gamma_b}}\partial'^i\partial'^j|\boldsymbol{x}_{\gamma_a}-\boldsymbol{x}'_{\gamma_b}|dA_{\gamma_a}dA'_{\gamma_b}. \tag{5.8}$$

Therefore the presence of matter with scalar dielectric or magnetic properties changes only the definition of the system parameters; they are increased by ϵ and μ factors. This is important because, in order to obtain great capacitances one can use suitable dielectrics: standard commercial types of capacitors can reach capacities of about 10 mF.

6. Application and Discussion

Let us now consider the device of Figure 1, located in the $x-y$ plane with $x = x^1$ and $y = x^2$, and evaluate its response to a periodic gravitational wave using realistic parameters. This circuit can be described as a set

of six conductors: two wires labelled γ_1 (the inductance and resistance) and γ_2 (idealized connection wire) which connect four plane conductors labelled with γ_3, γ_4 (to form the capacitor C_1) and γ_5, γ_6 (the capacitor C_2) with charge $Q_{\gamma_3} = -Q_1 + Q$, $Q_{\gamma_4} = Q_1 - Q$, $Q_{\gamma_5} = Q_2 + Q$ and $Q_{\gamma_6} = -Q_2 - Q$; Q_1 and Q_2 are the constant electrostatic charges on the plates of the two capacitors in the absence of gravitational waves when there is no current flow which satisfy

$$\begin{aligned} Q_1 + Q_2 =& Q_0, \\ Q_1/C_1 - Q_2/C_2 =& 0. \end{aligned} \tag{6.1}$$

The connection between the capacity of a condenser and the capacitance coefficients of the conductors i and j reads (see [15])

$$C^{-1} \equiv \hat{C}_{ii} - 2\hat{C}_{ij} + \hat{C}_{jj}. \tag{6.2}$$

The lagrangian of this system can be written as

$$\mathcal{L} = \frac{1}{2}L(t)\dot{Q}^2 - \frac{1}{2}\frac{(Q-Q_1)^2}{C_1(t)} - \frac{1}{2}\frac{(Q+Q_2)^2}{C_2(t)}. \tag{6.3}$$

Taking into account dissipation, the equation of the circuit becomes

$$L(t)\ddot{Q} + \dot{L}(t)\dot{Q} + R(t)\dot{Q} + \left(\frac{1}{C_1(t)} + \frac{1}{C_2(t)}\right) Q = \frac{Q_1}{C_1(t)} - \frac{Q_2}{C_2(t)}. \tag{6.4}$$

Let us now consider the case of a weakly coupled gravitational–wave field. Setting

$$\begin{aligned} \alpha_{+,\times} &= (\boldsymbol{e}_{+,\times})_{ij}\frac{\lambda^{ij}_{33}}{L} \\ \kappa_{+,\times} &= -(\boldsymbol{e}_{+,\times})_{ij}(\chi^{ij}_{11} - 2\chi^{ij}_{12} + \chi^{ij}_{22})\ C \\ \rho_{+,\times} &= (\boldsymbol{e}_{+,\times})_{ij}\frac{\varrho^{ij}_{3}}{R}, \end{aligned} \tag{6.5}$$

we can separate the polarization and angular dependences of the interaction (figure pattern). In this way we obtain

$$\begin{aligned} L(t) &= L[1 + \epsilon_L(t)] = L[1 + \alpha_+ h_+(t) + \alpha_\times h_\times(t)], \\ \frac{1}{C(t)} &= \frac{1}{C}[1 - \epsilon_C(t)] = \frac{1}{C}[1 - \kappa_+ h_+(t) - \kappa_\times h_\times(t)], \\ R(t) &= R[1 + \epsilon_R(t)] = R[1 + \rho_+ h_+(t) + \rho_\times h_\times(t)]. \end{aligned} \tag{6.6}$$

For the sake of simplicity let us put

$$\begin{aligned} C_1 = aC \qquad\qquad & C_2 = C \\ \omega_0^2 = & \frac{a+1}{a}\frac{1}{LC}. \end{aligned} \tag{6.7}$$

With this definition of the factor a we obtain

$$Q_1 = \frac{a}{a+1} Q_0 \qquad\qquad Q_2 = \frac{1}{a+1} Q_0 \tag{6.8}$$

and the equation of the circuit can be written as

$$\begin{aligned} (1+\epsilon_L(t))\ddot{Q} + 2\gamma \left(1 + \frac{\dot{\epsilon}_L(t)}{2\gamma} + \epsilon_R(t)\right)\dot{Q} + \omega_0^2 \left(1 - \frac{\epsilon_{C_1}(t) + a\epsilon_{C_2}(t)}{a+1}\right) Q = \\ = \omega_0^2 Q_0 \frac{a(\epsilon_{C_1}(t) - \epsilon_{C_2}(t))}{(a+1)^2}. \end{aligned} \tag{6.9}$$

As the time dependent coefficients in the l.h.s. are very small compared with unity their unique effect will be an amplitude and frequency modulation of the unperturbed charge motion

$$\ddot{Q} + 2\gamma\dot{Q} + \omega_0^2 Q = v_+ h_+(t) + v_\times h_\times(t) \tag{6.10}$$

where

$$v_{+,\times} = \omega_0^2 Q_0 \frac{a(\kappa_{1,+,\times} - \kappa_{2,+,\times})}{(a+1)^2}. \tag{6.11}$$

In equation (6.10) the interaction between the gravitational wave and the circuit is non–parametric. The dropped modulations are so small that they cannot be observed being a second order effect with respect to the solution of (6.10). If we consider a periodic gravitational wave $h_{+,\times}(t) = A_{+,\times}\ \cos(\omega_g t + \phi_{+,\times})$, then the solution of (6.10) can be written as

$$Q(t) = \ Q_+(t) + Q_\times(t) \tag{6.12}$$

$$Q_{+,\times}(t) = \frac{a(\kappa_{1,+,\times} - \kappa_{2,+,\times})}{(a+1)^2} \frac{2A_{+,\times} Q_0 \mathcal{Q}}{\sqrt{1 + \frac{(\omega_0^2 - \omega_g^2)^2}{4\gamma^2\omega_g^2}}} \frac{\omega_0}{\omega_g}\ \sin(\omega_g t + \phi_{+,\times} + \delta)$$

where $\mathcal{Q} = \omega_0/4\gamma$ is the quality factor of the circuit and

$$\delta = \arctan \frac{\omega_0^2 - \omega_g^2}{2\gamma\omega_g}. \tag{6.13}$$

In this system the effect of the gravitational wave is to cause a current $I(t)$ to flow in the circuit with

$$I(t) = \frac{a(\kappa_1 - \kappa_2)}{(a+1)^2} \frac{2A\omega_0 Q_0 \mathcal{Q}}{\sqrt{1 + \frac{(\omega_0^2-\omega_g^2)^2}{4\gamma^2\omega_g^2}}} \cos\left(\omega_g t + \phi + \delta\right). \tag{6.14}$$

Near resonance (i.e. $(\omega_0 - \omega_g)^2 << (2\gamma)^2$) we have

$$I_{near\ res}(t) = \frac{a(\kappa_1 - \kappa_2)}{(a+1)^2} \frac{2A\omega_0 Q_0 \mathcal{Q}}{1 + 2\left(\frac{\omega_0-\omega_g}{2\gamma}\right)^2} \cos\left(\omega_g t + \phi + \delta_{near\ res}\right) \tag{6.15}$$

where $\delta_{near\ res} \simeq \frac{\omega_0-\omega_g}{\gamma}$. This device behaves formally like a Weber bar with the initial static charge Q_0 corresponding to the rest length of the bar. The device has two advantages: firstly, it can be easily tuned to receive radiation from periodic sources like pulsars and binary stars (ν_g > few Hz); and secondly, the measurement is a null measurement (the expected current versus zero).

Let us now consider the superconducting case. Using the same notations eq. (6.4) becomes

$$L(t)\ddot{Q} + \dot{L}(t)\dot{Q} + R_s(t)\dot{Q}_n + \left(\frac{1}{C_1(t)} + \frac{1}{C_2(t)}\right) Q = \frac{Q_1}{C_1(t)} - \frac{Q_2}{C_2(t)}. \tag{6.16}$$

For weak gravitational waves we have

$$\begin{aligned}(1+\epsilon_L(t))\ddot{Q} + \dot{\epsilon}_L(t)\dot{Q} + 2\gamma_s[1+\epsilon_R(t)]\dot{Q}_n + \omega_0^2\left(1 - \frac{\epsilon_{C_1}(t) + a\epsilon_{C_2}(t)}{a+1}\right) Q = \\ = \omega_0^2 Q_0 \frac{a(\epsilon_{C_1}(t) - \epsilon_{C_2}(t))}{(a+1)^2}.\end{aligned} \tag{6.17}$$

Again the time dependent coefficients in the l.h.s. of equation (6.17) produce only a very small frequency and amplitude modulation of the solution of the following equation

$$\ddot{Q} + 2\gamma_s\dot{Q}_n + \omega_0^2 Q = v_+ h_+(t) + v_\times h_\times(t) \tag{6.18}$$

where

$$v_{+,\times} = \Omega_0^2(1+2\Gamma)Q_0 \frac{a(\kappa_{1,+,\times} - \kappa_{2,+,\times})}{(a+1)^2}. \tag{6.19}$$

In equation (6.18) the gravitational–wave electromagnetic–circuit interaction is purely non–parametric. If we use relation (4.28) in (6.18) we find the equation of the supercurrent as

$$\dddot{Q}_s + \beta(1+2\Gamma)\ddot{Q}_s + \Omega_0^2(1+2\Gamma)\dot{Q}_s + \beta\Omega_0^2(1+2\Gamma)Q_s = \beta[v_+ h_+(t) + v_\times h_\times(t)]. \tag{6.20}$$

If $h_{+,\times}(t) = A_{+,\times}\ \cos(\omega_g t + \phi_{+,\times})$, then the solution of (6.20) can be written as

$$Q_s(t) = Q_{s,+}(t) + Q_{s,\times}(t) \tag{6.21}$$

$$Q_{s,+,\times}(t) = \frac{a(\kappa_{1,+,\times} - \kappa_{2,+,\times})}{(a+1)^2} \frac{2A_{+,\times}(1+2\Gamma)Q_0 \mathcal{Q}_{eff}}{T(\omega_g)} \frac{\Omega_0}{\omega_g}\ \sin(\omega_g t + \phi_{+,\times} + \delta)$$

where

$$\tan\delta = \frac{(1+2\Gamma)(\Omega_0^2 - \omega_g^2)}{2\gamma_0\omega_g\left(1 + \frac{\Omega_0^2 - \omega_g^2}{2\gamma_0\beta}\right)}, \tag{6.22}$$

$$T(\omega_g) = \left(1 + \frac{\Omega_0^2 - \omega_g^2}{2\gamma_0\beta}\right)\sqrt{1+\tan^2\delta} \tag{6.23}$$

and

$$\mathcal{Q}_{eff} = \frac{\Omega_0}{4\gamma_0} = \frac{\beta}{4\Gamma\Omega_0} = \frac{\beta^2}{2\Omega_0}\frac{L}{R_s} \tag{6.24}$$

is the quality factor of the superconducting circuit which is several orders of magnitude greater than that of the ohmic circuit. In this way the gravitational wave gives rise to a supercurrent

$$I_s(t) = \frac{a(\kappa_1 - \kappa_2)}{(a+1)^2} \frac{2A(1+2\Gamma)\Omega_0 Q_0 \mathcal{Q}_{eff}}{T(\omega_g)}\ \cos(\omega_g t + \phi + \delta) \tag{6.25}$$

in which, for the sake of simplicity, we have considered only one polarization state and dropped the indices $+, \times$. Near resonance (i. e. $(\Omega_0^2 - \omega_g^2)^2 << (2\gamma_0)^2$)

we have

$$I_{s\ near\ res}(t) = \frac{a(\kappa_1 - \kappa_2)}{(a+1)^2} \frac{2A\Omega_0 Q_0 \mathcal{Q}_{eff}}{1 + 2\left(\frac{\Omega_0 - \omega_g}{2\gamma_0}\right)^2} \cos(\omega_g t + \phi + \delta_{near\ res}) \quad (6.26)$$

where $\delta_{near\ res} \simeq \frac{\Omega_0 - \omega_g}{\gamma_0}$. To get the total current we have to use (4.28) for the normal current,

$$I_n(t) = -\frac{\omega_g}{\beta} \frac{a(\kappa_1 - \kappa_2)}{(a+1)^2} \frac{2A(1+2\Gamma)\Omega_0 Q_0 \mathcal{Q}_{eff}}{T(\omega_g)} \sin(\omega_g t + \phi + \delta). \quad (6.27)$$

We see that this current (in agreement with (4.4) and (4.5)) is smaller by a factor ω_g/β and it is $\pi/2$ out of phase with respect to the supercurrent. The response of a superconducting device differs from the response in the ohmic case because of different quality factors.

Let us now calculate explicitly the parameters of the circuit. The eqs. (3.22-24) and (3.18-20) or (5.5-6) and (5.7-8) give:

$$\begin{gathered} \delta_{ij} \lambda^{ij}_{\gamma_a \gamma_b} = 2L_{\gamma_a \gamma_b} \\ \delta_{ij} Q^{\gamma_a} \chi^{ij}_{\gamma_a \gamma_b} Q^{\gamma_b} = Q^{\gamma_a} \hat{C}_{\gamma_a \gamma_b} Q^{\gamma_b} \quad \Rightarrow \quad \delta_{ij} \chi^{ij}_{\gamma_a \gamma_b} = \hat{C}_{\gamma_a \gamma_b} \\ \delta_{ij} \rho^{ij}_{\gamma_a} = R_{\gamma_a} \end{gathered} \quad (6.28)$$

We note that

$$\partial'^1 \partial'^1 |\boldsymbol{x}_{\gamma_a} - \boldsymbol{x}'_{\gamma_b}| = \frac{(y-y')^2 + (z-z')^2}{|\boldsymbol{x}_{\gamma_a} - \boldsymbol{x}'_{\gamma_b}|^3}, etc. \quad (6.29)$$

so that

$$Q^{\gamma_a} \chi^{ii}_{\gamma_a \gamma_b} Q^{\gamma_b} = \frac{1}{2} Q^{\gamma_a} \hat{C}_{\gamma_a \gamma_b} Q^{\gamma_b} - \frac{1}{2\epsilon} \int_{\gamma_a} \int_{\gamma_b} \frac{dQ^{\gamma_a}}{dA_{\gamma_a}} \frac{dQ^{\gamma_b}}{dA'_{\gamma_b}} \frac{(x^i_{\gamma_a} - x'^i_{\gamma_b})^2}{|\boldsymbol{x}_{\gamma_a} - \boldsymbol{x}'_{\gamma_b}|^3} dA_{\gamma_a} dA'_{\gamma_b} \quad (6.30)$$

(no summation over i).

Let us now consider the case of a capacitor made of two circular plates set perpendicular to the x axis and at a distance d apart. If $\gamma_a = \gamma_b$ then

$$\chi^{11}_{\gamma_a \gamma_a} = \frac{1}{2} \hat{C}_{\gamma_a \gamma_a} \quad (6.31)$$

and

$$\chi^{22}_{\gamma_a\gamma_a} = \chi^{33}_{\gamma_a\gamma_a} = \frac{1}{4}\hat{C}_{\gamma_a\gamma_a}. \tag{6.32}$$

If $\gamma_a \neq \gamma_b$, assuming that d is much smaller than the diameter of the circular plates, by means of (6.28) and the symmetry of the problem, we approximately get

$$\begin{aligned} \chi^{11}_{\gamma_a\gamma_b} &= \hat{C}_{\gamma_a\gamma_b} - \frac{1}{2}\hat{C}_{\gamma_a\gamma_a} \\ \chi^{22}_{\gamma_a\gamma_b} = \chi^{33}_{\gamma_a\gamma_b} &= \frac{1}{4}\hat{C}_{\gamma_a\gamma_a} \end{aligned} \tag{6.33}$$

which is valid also for $\gamma_a = \gamma_b$ as can be seen by comparing (6.33) with (6.31–32). Similarly one can show that $\chi^{12}_{\gamma_a\gamma_b} = 0$.

Now let us assume that the wave comes from the $z = x^3$ axis perpendicular to the plane containing our circuit (see Figure 1). In this case we have for the polarization tensors

$$(e_+)_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{6.34}$$

$$(e_\times)_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{6.35}$$

With reference to eq. (6.5) let us calculate κ_+; one finds

$$\begin{aligned} \kappa_+ &= -(e_+)_{ij}\left(\chi^{ij}_{11} - 2\chi^{ij}_{12} + \chi^{ij}_{22}\right)C = \\ &= \left[\left(\chi^{22}_{11} - \chi^{11}_{11}\right) - 2\left(\chi^{22}_{12} - \chi^{11}_{12}\right) + \left(\chi^{22}_{22} - \chi^{11}_{22}\right)\right]C. \end{aligned} \tag{6.36}$$

Taking the x axis parallel to the plates of C_1 and the y axis parallel to the plates of C_2 we obtain

$$\begin{aligned} (\kappa_+)_1 &= 1, \qquad & (\kappa_\times)_1 &= 0 \\ (\kappa_+)_2 &= -1, \qquad & (\kappa_\times)_2 &= 0 \end{aligned} \tag{6.37}$$

Substituting (6.37) in (6.15) and (6.26), and taking $a = 1$ one gets

$$\begin{aligned} I_{res} &= A_+\omega_0 Q_0 \mathcal{Q}\cos(\omega_0 t + \phi), \\ I_{res.s} &= A_+\Omega_0 Q_0 \mathcal{Q}_{eff}\cos(\omega_0 t + \phi). \end{aligned} \tag{6.38}$$

We now estimate the minimal gravitational amplitude which can be detected by our device. Once the device is isolated from the outside world by means for instance of mechanical filters, as those used in gravitational bar detectors and Faraday cages, the main noise source is the thermal one, as well for the mechanical part of the detector as for the electric part, e.g. the back–action noise of the amplifier, at cryogenic temperature and for the best amplifiers, is at most of the order of magnitude of the thermal noise. In the following we shall derive formulae for the thermal noise in the electric part of the detector. The thermal spectral noise in the mechanical part ($2\delta l/l = h$, see equation (2.17)) is given by $\sqrt{16k_BT/M\tau l^2\omega^4}$ where T, M, τ, and l denote temperature, effective mass, life-time of the fundamental mode, and effective length of the detector, k_B is the Boltzmann constant and ω denotes the angular frequency of the gravitational wave, e.g. see [24]. Assuming the numbers $T = 4$ K, $M = 10^4$ kg, $\tau = 10$ sec, $l = 10$ m, and $\omega/2\pi = 60$ Hz yields about 2.1×10^{-23} for h, measured through a period of 4 months. This number is as small as the smallest number in the equation (6.45) below, i.e. for the superconducting circuits in question the mechanical part has to be adjusted to those numbers.

We now make the hypothesis that the only resistance in our circuit is that of our scheme in Figure 1. The mean square fluctuation of the voltage noise is therefore given for $t_{obs} >> \mathcal{Q}/\omega_0$ by ([25] p.288)

$$< V_n^2 >= \frac{k_BT}{C_{eff}} \tag{6.39}$$

where C_{eff} is the total capacitance of the capacitors. The mean noise energy is

$$C_{eff} < V_n^2 >= k_BT \tag{6.40}$$

The energy dissipated by the ohmic and superconducting current produced by

the gravitational wave is

$$\begin{aligned} W &= \int R I^2 dt = \frac{1}{2} R I_0^2 t_{obs} \\ W_s &= \int R_s I_n^2 dt = \frac{1}{2} R_s I_{n0}^2 t_{obs} \end{aligned} \tag{6.41}$$

where t_{obs} is the observational time and I_0 the amplitude of the current I. Equating (6.40) and (6.41) one finds the minimum detectable current after an observation time t_{obs},

$$\begin{aligned} I_0 &= \sqrt{\frac{2k_B\ T}{R\ t_{obs}}} \\ I_{n0} &= \sqrt{\frac{2k_B\ T}{R_s\ t_{obs}}} \end{aligned} \tag{6.42}$$

At resonance this implies (by means of eqs. (6.7), (6.15) and $\mathcal{Q} = \omega_0 L/(2R)$ for the ohmic circuit and (4.22), (6.24) and (6.27) for the superconducting one) that

$$\begin{aligned} A_{noise} &= \frac{1}{V_0}\ \sqrt{\frac{(a+1)^4}{a^2(\kappa_1-\kappa_2)^2}\frac{k_B T}{\mathcal{Q} C\omega_0 t_{obs}}} \\ A_{noise\ s} &= \frac{1}{V_0}\ \sqrt{\frac{(a+1)^4}{a^2(\kappa_1-\kappa_2)^2}\frac{k_B T}{\mathcal{Q}_{eff} C\omega_0 t_{obs}}} \end{aligned} \tag{6.43}$$

where $V_0 = Q_0/C$. If $a = \kappa_1 = -\kappa_2 = 1$ then

$$\begin{aligned} A_{noise} &= \frac{2}{V_0}\ \sqrt{\frac{k_B T}{\mathcal{Q} C\omega_0 t_{obs}}} \\ A_{noise\ s} &= \frac{2}{V_0}\ \sqrt{\frac{k_B T}{\mathcal{Q}_{eff} C\omega_0 t_{obs}}} \end{aligned} \tag{6.44}$$

Taking $V_0 = 10^5$ Volt, $T = 4$ K, $C = 10^{-2}$ F, $\omega_0 = 2\pi \times 60$ rad/sec, $t_{obs} = 4$ months $= 10^7$ sec, $\mathcal{Q} = 10^3$ and $\mathcal{Q}_{eff} = 10^6$ one has

$$A_{noise} \simeq 7.6 \times 10^{-22}, \qquad A_{noise\ s} \simeq 2.4 \times 10^{-23} \tag{6.45}$$

Finally we compute the order of magnitude of the normal and superconducting current, produced by a gravitational wave with amplitudes $A_+ = 2 \times 10^{-21}$ and

$A_+ = 5 \times 10^{-23}$ which correspond respectively to a signal to noise ratio equal to 2; we find

$$\begin{aligned} I_n &\simeq \ 7.5 \times 10^{-13}\ A \\ I_s &\simeq \ 2 \times 10^{-11}\ A \end{aligned} \tag{6.46}$$

that are within the possibilities of actual measurements with SQUIDs. If we go down with the temperature in the ultracryogenic zone (~ 40 mK) we obtain

$$\begin{aligned} A_{noise} &\simeq 7.6 \times 10^{-23} \qquad\qquad I \simeq 7.5 \times 10^{-14}\ A \\ A_{noise\ s} &\simeq 2.4 \times 10^{-24} \qquad\qquad I_s \simeq 2 \times 10^{-12}\ A \end{aligned} \tag{6.47}$$

Further improvements can come either from different arrangements of the circuit elements (for instance one can consider a series of many capacitors). As far as the last question is concerned, today's limitations in the superconducting case are due to the energy dissipation in the material which acts as a mounting of the circuit. For this reason we have set $\mathcal{Q}_{eff} = 10^6$ but there are no theoretical limitations in thinking of quality factors higher by several orders of magnitude: it is in fact only a technological problem. We think that a serious research in this field would be very useful.

It is to be expected that when the device is operating near the mechanical resonance the sensitivity should increase. However this situation deserves a more detailed investigation.

Recall now that we have calculated the currents in TT–gauge. In actual experiments we measure currents in the laboratory reference frame (FNC). Because of the charge conservation we can write

$$I^{FNC}\ dy^0 \ = \ I^{TT}\ dx^0 \tag{6.48}$$

where y^μ are the FNC coordinates while x^μ are the TT coordinates. As $x^0 = y^0 - \frac{1}{4}h_{ij}{}^{,0}y^i y^j$ we have

$$dx^0 = \left(1 - \frac{k^2}{4}h_{ij}^{(2)}y^i y^j\right) dy^0 - \frac{k}{2}h_{ij}^{(1)}y^i dy^j \tag{6.49}$$

where the superscript (i) is the i–th derivative of h with respect to its argument. Therefore the relation between the current in the two gauges reads

$$I^{FNC} = I^{TT}\left(1 - \frac{k^2}{4}h_{ij}^{(2)}y^iy^j - \frac{k}{2}h_{ij}^{(1)}y^i\frac{dy^j}{dy^0}\right). \tag{6.50}$$

In our approximation both first order corrections are negligible and so

$$I^{FNC} \;=\; I^{TT} \tag{6.51}$$

We point out that our theoretical limits in the magnitude of the gravitational–wave amplitude detectable from periodic sources like the Crab pulsar are better than those experimentally obtained by a Japanese research group ($A < 5 \times 10^{-22}$ [20]; see also [21]). Therefore, a superconducting device at liquid helium temperature could give new limits for the emission of gravitational waves from this pulsar. As to vary an electromagnetic frequency is much simpler than varying a mechanical one, our detectors can be easily adjusted to any gravitational–wave frequency measurable on Earth.

References

1. G. A. Lupanov, Sov. Phys. JETP **25**, 76 (1967); F. Cooperstock, Ann. Phys. (N.Y.) **47**, 173 (1968); D. Boccaletti, V. De Sabbata, P. Fortini and C. Gualdi, Nuovo Cimento **B 70**, 129 (1970); P. Fortini, C. Gualdi and A. Ortolan, Nuovo Cimento **B 106**, 395 (1991).
2. K. S. Thorne in *Three hundred years of gravitation*; eds. S. Hawking and W. Israel, Cambridge Univ. Press, Cambridge (1987).
3. V. B. Braginsky, M. B. Menskii Sov. Phys. JEPT Lett. **13**, 417 (1971); V. B. Braginsky, L. P. Grishchuk, A. G. Doroskevich, Ya. B. Zel'dovich, I. D. Novikov and M. V. Sazhin Sov. Phys. JETP **38**, 865 (1973); F. Pegoraro, E. Picasso and L. A. Radicati J. of Phys. **A 11**, 1949 (1978); C. M. Caves Phys. Lett. **B 80**, 393 (1979).
4. C. W. Misner, K. S. Thorne and J. A. Wheeler *Gravitation*; Freeman, San Francisco (1973).
5. P. Fortini and C. Gualdi, Nuovo Cimento **B 71**, 37 (1982).
6. G. Callegari, P. Fortini and C. Gualdi, Nuovo Cimento **B 100**, 421 (1987).
7. S. Weinberg, *Gravitation and Cosmology*; Wiley, New York (1972) Ch. 5.
8. R. C. Tolman, *Relativity, Thermodynamics and Cosmology*; Clarendon Press, Oxford (1966).
9. V. N. Golubenkov and Ya. A. Smorodinsky, Zh. eksper. teor. Fiz. **31**, 330 (1956).
10. G. Schäfer and H. Dehnen, J. Phys. **A 13**, 2703 (1980).
11. B. Mours and M. Yvert, Phys. Lett. **A 136**, 209 (1989).
12. F. London, Superfluids, Dover Publ. (1961); F. London Phys. Rev. **51**, 678(L) (1937).
13. H. Goldstein, *Classical Mechanics*, second edition, Addison Wesley (1980).

14. L. D. Landau, E. M. Lifshitz, *Course of Theoretical Physics Vol. 2: The Classical Theory of Fields*, Pergamon Press (1975).

15. L. D. Landau, E. M. Lifshitz, *Course of Theoretical Physics Vol. 8: Electrodynamics of Continuous Media*, Pergamon Press (1960).

16. R. Schrader Phys. Lett. **B 143**, 421 (1984).

17. J. L. Stone and W. H. Hartwig J. Appl. Phys. **39**, 2665 (1968).

18. McLachlan, *Theory and Application of Mathieu Functions*, Clarendon Press (1947).

19. L. D. Landau, E. M. Lifshitz, *Course of Theoretical Physics Vol. 1: Mechanics*, Pergamon Press (1960).

20. S. Owa, M. K. Fujimoto, H. Hirakawa, K. Morimoto, T. Suzuki, K. Tsubono, Proc. IV Marcel Grossmann meeting on Gen.Rel. p. 571-8, Hu Ning Ed. North Holl. (1986).

21. H. Hirakawa, K. Narihara Phys. Rev. Lett. **35**, 330 (1975); H. Hirakawa, K. Narihara, M. K. Fujimoto J. Phys. Soc. Jap. **41**, 1093 (1976); T. Suzuki, K. Tsubono, H. Hirakawa Phys. Lett. **A 67**, 2 (1978); K. Oide, H. Hirakawa Phys. Rev. **D 20**, 2480 (1979); S. Kimura, T. Suzuki, H. Hirakawa Phys. Lett. **A 81**, 302 (1981); M. K. Fujimoto Proc. III Marcel Grossmann meeting on Gen.Rel. p. 1447, Hu Ning Ed. North Holl.(1983); K. Kuroda, H. Hirakawa Japan. J. Phys. **22**, 1005 (1983); K. Kuroda, K. Tsubono, H. Hirakawa Jap. Jour. Appl. Phys. **23**, L415 (1984); K. Tsubono, M. Ohashi, H. Hirakawa Jap. Jour. Appl. Phys. **25**, 622 (1986); Y. Nagashima, S. Owa, K. Tsubono, H. Hirakawa Rev. Sci. Instrum. **59**, 112-4 (1988).

22. K. S. Thorne in *Gravitational Radiation*; eds. N. Deruelle and T. Piran, North–Holland Publishing, Amsterdam (1983) p. 1.

23. L. Baroni, G. Callegari, P. Fortini, C. Gualdi, M. Orlandini in *Proceedings*

of the Fourth Marcel Grossmann Meeting on General Relativity; ed. R. Ruffini (Elsevier Science Publ., Amsterdam 1986) p. 641.

24. J. Colas, L. Massonnet, B. Mours and M. Yvert, *Détecteur d'ondes gravitationnelles capacitif et bruit thermique*, LAPP–EXP–90–01, May 1990.

25. A. Papoulis, *Probability, random variables and stochastic processes* - Second Edition - McGraw–Hill, New York (1984).

Figures

Fig.1 — A scheme of the RLC circuit describrd in the text which can be used as a detector of gravitational waves. A static charge is distributed on the plates of the condensers C_1 and C_2.

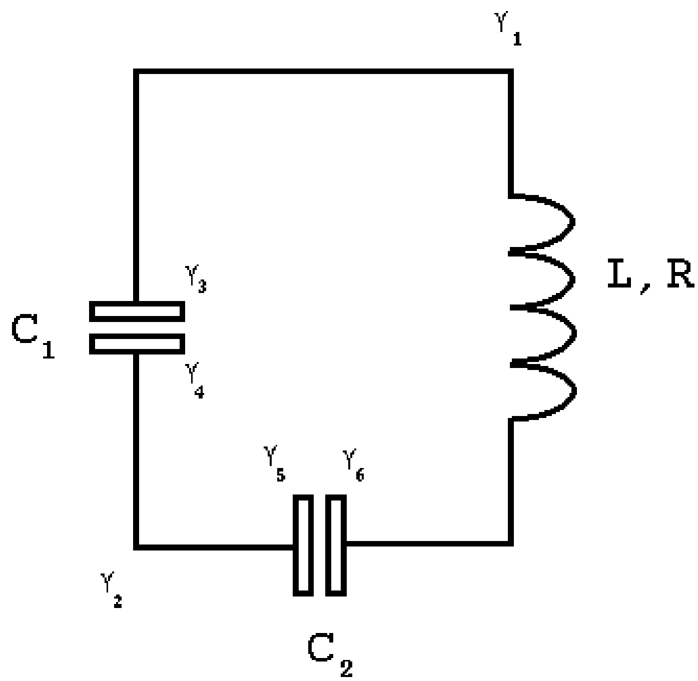

Fig. 1